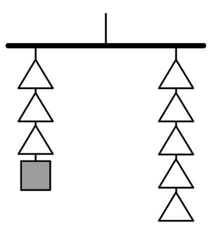
Unit 4 Summary

Prior Learning	Grade 8, Unit 4	Later in Grade 8	High School
Grade 8, Unit 3 • Writing linear equations, such as $y = 2x + 3$ • Graphing equations • Slope and y-intercept	• Solve equations in one variable, such as $3x + 20 = 7x$. • Solve systems of two linear equations using graphs and symbols.	Unit 5 • Use equations to describe functions.	• Solve nonlinear equations. • Solve systems of more than two equations.

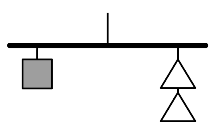
Solving Linear Equations

Solving an equation means finding all values that make the equation true.

$x = 2$ is a solution to the equation $3x = 6$ because $3(2) = 6$.



$3x+5=5x$



$5=2x$

A true equation is like a balanced hanger—if you perform the same operations to both sides, the hanger remains balanced.

The equations $3x + 5 = 5x$ and $5 = 2x$ are equivalent because we subtracted $3x$ (removed three triangles) from both sides.

When an equation requires several operations in order to determine a solution, we write each equation on its own line.

Here we use the **distributive property**: Add $6x$, subtract 2, and divide by 11 to both sides of the equation to determine a solution.

$$
\begin{aligned}
2(-3x+4) &= 5x+2 \\
-6x+8 &= 5x+2 \\
8 &= 11x+2 \\
6 &= 11x \\
\frac{6}{11} &= x
\end{aligned}
$$

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Systems of Linear Equations

A system of equations is a set of two (or more) equations where the variables represent the same values.

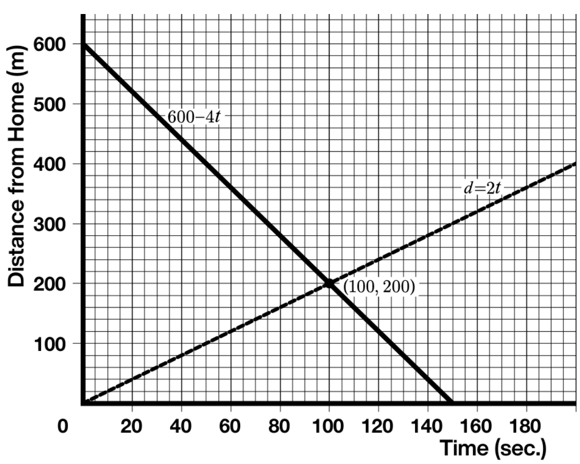
Solving a system of equations means finding values for the variables that make both equations true.

$$y = 2x + 5$$

$$y = 3x + 1$$

Here is an example of a situation where systems of equations are useful:

Yona is running home from school at 4 meters per second. Her brother Haruto is walking to school from home at 2 meters per second. They leave at the same time and their school is 600 meters from their home. When will Yona and Haruto pass each other? How far will they be from home?



If you write an equation for each child's distance from home, the two equations form a system:

Yona: $d = 600 - 4t$

Haruto: $d = 2t$

The solution to the system is the point where the lines cross on the graph.

The question asks when the distances will be equal, so you can set these expressions equal to each other and solve for the time.

$$600 - 4t = 2t$$
$$600 = 6t$$
$$100 = t$$

Once you know the time, use the equations to find the children's distances at that time.

Yona and Haruto pass each other when 100 seconds have passed. They are 200 meters from home.

$$d = 600 - 4(100) \qquad d = 2(100)$$
$$d = 600 - 400 \qquad d = 200$$
$$d = 200$$

Try This at Home

Solving Linear Equations

1. Solve this equation: $3(3-3x)=2(x+3)-30$

2. Elena and Noah work on the equation $\frac{1}{2}(x+4)=-10+2x$ together.

   Here is their work:

   Do you agree with their solutions? Explain or show your reasoning.

Elena:

$$\frac{1}{2}(x+4)=-10+2x$$
$$(x+4)=-20+2x$$
$$x+24=2x$$
$$24=x$$
$$x=24$$

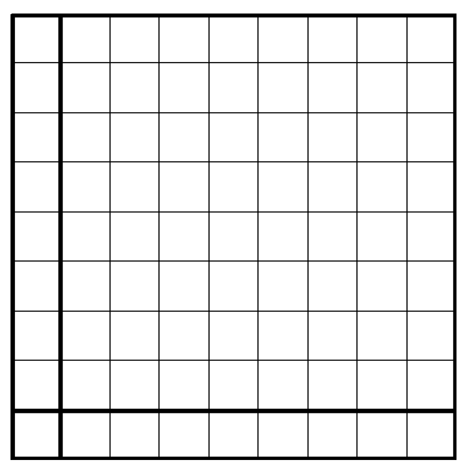
Noah:

$$\frac{1}{2}(x+4)=-10+2x$$
$$x+4=-20+2x$$
$$-3x+4=-20$$
$$-3x=-24$$
$$x=-8$$

Systems of Linear Equations

Tiam and Maneli are biking in the same direction on the same path, but they start at different times. Tiam is riding at a constant speed of 18 miles per hour. Maneli started riding at a constant speed of 12 miles per hour a quarter of an hour (15 minutes) before Tiam started.

3.1 Write equations to represent the relationship between time and distance biked for each person.



3.2 Graph both equations on the same set of axes.

3.3 Use the equations and/or the graph to find the time and distance that Tiam and Maneli meet.

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Solutions:

1. There are many possible ways to solve this equation, all with a correct solution of $x = 3$.

Here is one example:

$$3(3 - 3x) = 2(x + 3) - 30$$
$$9 - 9x = 2x + 6 - 30$$
$$9 - 9x = 2x - 24$$
$$-9x = 2x - 33$$
$$-11x = -33$$
$$x = 3$$

2. No, they both have errors in their solutions.
   - Elena multiplied both sides of the equation by 2 in her first step. She did not multiply the $2x$ by the 2. The second line should be $(x + 4) = -20 + 4x$.
   - We can check Elena's solution by replacing x with 24 in the original equation to see if the equation is true. Since 14 is not equal to 38, Elena's solution is not correct.

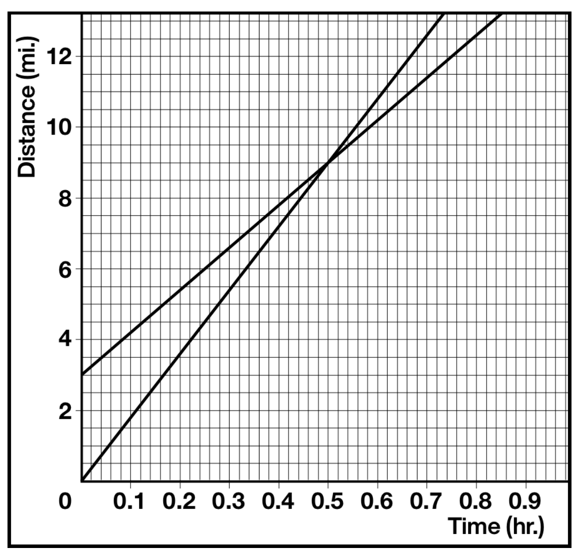
$$\frac{1}{2}(x+4) = -10+2x$$
$$\frac{1}{2}(24+4) = -10+2(24)$$
$$\frac{1}{2}(28) = -10+48$$
$$14 = 38$$

   - Noah divided both sides in his last step. He wrote -8 as the quotient on the right hand side instead of 8. $\frac{-24}{-3} = 8$. His last line should be $x = 8$.
   - We can also check Noah's solution by replacing x with -8 in the original equation to see if the equation is true. Noah's solution is not correct.

3.1 **Tiam:** $d = 18t$

Maneli: $d = 12\left(t + \frac{1}{4}\right)$ or $d = 12t + 3$

3.2 See the graph on the right.



3.3 Using the graph, Tiam and Maneli are at the same time and distance when their graphs cross, which is 0.5 hours since Tiam started riding (0.75 hours since Maneli started riding). They meet after having biked 9 miles.

Using equations, you can set the two expressions equal to each other and write the equation $18t = 12\left(t + \frac{1}{4}\right)$.

One way to solve this equation is shown on the right.

First, use the distributive property to rewrite the right-hand side of the equation.

Then, subtract $12t$ from both sides.

Finally, divide both sides of the equation by 6.

$$18t = 12\left(t + \frac{1}{4}\right)$$

$$6t = 3$$

$$t = \frac{1}{2}$$

After a half hour, Tiam and Maneli have each ridden 9 miles. Both strategies—using the graph and the equations—give the same result.